True False 1

## True/False - 1

Is the following statement true, false, or uncertain/need more information:

Arithmetic average return is always greater than or equal to geometric average return.

- ○ True
- ○ Uncertain/need more information
- ○ False

Arithmetic mean always greater than or equal to geometric mean. Equal sign applies if and only if all the numbers (returns) in the series have the same value.

True False 2

## True/False - 2

Is the following statement true, false, or uncertain/need more information:

Nominal interest rate is always higher than real interest rate.

- ○ False
- ○ True
- ○ Uncertain/need more information

Nominal rate equals real rate plus expected inflation. Expected inflation is not always positive. For example, in a recession, the value of currency can decline, which is called deflation.

True False 3

## True/False - 3

Is the following statement true, false, or uncertain/need more information:

If your certainty equivalence of a gamble is less than its probabilistically fair value, then you must have a concave utility function.

- ○ True
- ○ False
- ○ Uncertain/need more information

The following are equivalent:

Investor has a concave utility function;

Investor is risk-averse;

Certainty equivalence is less than expected payoff (probabilistically fair value);

Insurance premium is positive.

True False 4

## True/False - 4

Is the following statement true, false, or uncertain/need more information:

Most ETFs are riskier investment compared to most hedge funds.

- ○ Uncertain/need more information
- ○ True
- ○ False

Most ETFs have very low risk because they are passive investment and follow a broad index. Some hedge funds can have relatively low risk but most hedge funds are high-risk investment.

True False 5

## True/False - 5

Is the following statement true, false, or uncertain/need more information:

During the period from 1926 to 2020, the risk premium for large-cap U.S. stocks is higher than the risk premium for small-cap U.S. stocks on average.

- ○ False
- ○ Uncertain/need more information
- ○ True

Small cap stocks are inherently more risky, which commands a higher risk premium.

Short Squeeze Part 1

## Short squeeze - part 1

The following questions are about applying your knowledge of security trading in understanding short squeeze. Short squeeze describes a phenomenon where short sellers are forced to close their short position due to lack of liquidity and create a cascading effect on stock price. When it happens, a mild initial price movement can trigger a domino effect that pushes the stock price drastically in one direction. The GameStop episode earlier this year is likely a short squeeze event.

Imagine stock ABC trades at $170 now. The market maker place a depth of exactly 1 share at each cent. That is, if you trade 1 share of stock ABC, its price moves 1 cent. You decide to buy 300 shares of ABC. After your trade, what's the price at which stock ABC trades?

$173. If you buy stock, you lift the ask quotes and push the price up.

Short Squeeze Part 2

## Short squeeze - part 2

Now imagine there exists a short seller, Janet, who has a short position on stock ABC. Janet started her short operation when the stock price was $153. When initiating the short operation, Janet had $232,086 worth of Treasury Bill in her brokerage account that can be used as collateral. The broker requires an initial margin of 60% and a maintenance margin of 42% and also requires that Janet sets aside 100% of the cash proceed as collateral. Janet shorted as much as she can given the constraints. What's the threshold price at which Janet is going to receive a margin call from her broker? (Fill in a number with 2 decimals.)

We've done this to death. $172.40.

Short Squeeze Part 3

## Short squeeze - part 3

Assume Janet doesn't have any additional capital so that when her broker issues a margin call, she is forced to close out the short operation. When closing out a short operation, the short seller needs to buy back the stock she shorted from the market and return it to her broker (because the short seller borrowed it from the broker when she initiated the operation).

Now imagine the same scenario (from your perspective) in part 1 of the question. The current price of stock ABC is $170 and you buy 300 shares. Knowing the existence of Janet's short position, what's the price at which stock ABC trades after all possible trades execute? (Fill in a number with 2 decimal points.)

Hint: If your trade triggers the margin call for Janet, she will be forced to close out her short position.

Because your buy order pushes the price over the threshold, therefore triggers Janet's margin call and forces her to clear her position. In doing so, Janet has to buy which further pushes up the stock price. Janet shorted 2528 shares, so the price will be pushed up to $198.28.

Short Squeeze Part 4

# Short squeeze - part 4

Now imagine in addition to Janet, there are three more short sellers, Jay, Ben, and Alan, who have outstanding short positions. The following table describes their short operations:

| Short Seller | Stock Price when Initiating Short | T-bill as Collateral | Initial Margin | Maintenance Margin |
|---|---|---|---|---|
| Janet | $153 | $232,086 | 60% | 42% |
| Jay | $160 | $135,936 | 55% | 35% |
| Ben | $148 | $304,762 | 70% | 25% |
| Alan | $168 | $174,260 | 80% | 20% |

All four of them shorted as much as they can when they initiated the operation. Also, none of them have spare liquidity at hand, i.e. if they face a margin call, they will be forced to close out their short position.

The current price of stock ABC is $170 and you decide to buy 300 shares. Now what the stock price should end up being? (Fill in a number with 2 decimal points.)

Hint: Use your logic in last question to identify whose short positions is/are forced to be closed out.

Comment: Notice how much difference between part 1 and part 4, even though the trigger is all the same - you buying merely 300 shares. The mechanism portraited in this question describes the phenomenon people refer to as "short squeeze".

Check if the new price $198.28 triggers any additional margin call for the short sellers. And repeat the process. The final price will be $243.13. The following two tables describe the detail.

| Entering price | Collateral | Initial Margin | Maintenance Margin | Shares Shorted | Cash Proceed | Total Asset | Threshold |
|---|---|---|---|---|---|---|---|
| 153 | 232086 | 60% | 42% | 2528 | 386784 | 618870 | 172.3987 |
| 160 | 135936 | 55% | 35% | 1544 | 247040 | 382976 | 183.7344 |
| 148 | 304762 | 70% | 25% | 2941 | 435268 | 740030 | 201.3002 |
| 168 | 174260 | 80% | 20% | 1296 | 217728 | 391988 | 252.0499 |

| Pre-trade Price | Shares traded | Post-trade Price | Trigger anything? |
|---|---|---|---|
| 170 | 300 | 173 | Yes |
| 173 | 2528 | 198.28 | Yes |
| 198.28 | 1544 | 213.72 | Yes |
| 213.72 | 2941 | 243.13 | No |

Capital Market Part 1

## Capital market - part 1

Assume that an ETF that tracks S&P 500 Index (market portfolio) is worth $40 today. In a year, its value will be $60 if the economy is booming whereas if we enter a recession, its value will drop to $30. In the meantime, there are two stocks, X and Y, also worth $40 today. In a good economy, X and Y will have prices of $51.44 and $33.76 whereas in a bad economy, X and Y will have prices of $35.60 and $47.97. It is equally likely to have a good or bad economy in the next year. The following table summarize the information:

| | Value today | Value in a year - good economy | Value in a year - bad economy |
|---|---|---|---|
| ETF (market portfolio) | $40.00 | $60.00 | $30.00 |
| Stock X | $40.00 | $51.44 | $35.60 |
| Stock Y | $40.00 | $33.76 | $47.97 |

What can you say about stock X and stock Y's betas? (Fill in positive, negative, or uncertain.)

Stock X has a [ ] beta and stock Y has a [ ] beta.

No calculation needed. You can eyeball the future value and see that stock X goes up when market goes up and goes down when market goes down, which means it has a positive beta. The opposite is true for stock Y, which has a negative beta.

Capital Market Part 2

## Capital market - part 2

Use all the relevant information from "capital market - part 1" for this question. What is the expected return of stock X and stock Y? (Fill in numbers with two decimal points. Also notice that the percentage sign % is already written outside of the blanks.)

Stock X has an expected return of [ ] % and stock Y has an expected return of [ ] %.

Calculate the expected value of stocks X (43.52) and Y (40.865) first and calculate the return. Stock X has an expected return of 8.80% and stock Y has an expected return of 2.16%.

Capital Market Part 3

## Capital market - part 3

Use all the relevant information from "capital market - part 1" for this question. You have a log utility function $U = \ln(w)$ where $w$ is your wealth. The only assets you have today are one share of the ETF and $40 cash. You decide to hold on to the ETF and try to make a decision of what to do with the $40 cash. You can buy one share of stock X, buy one share of stock Y, or keep it as cash (which earns 0% interest). What is your respective expected utility given different investment decision? (Fill in numbers with 6 decimal points.)

If you invest in stock X, your expected utility will be [ ] ; if you invest in stock Y, your expected utility will be [ ] ; and if you keep it as cash, your expected utility will be [ ] .

It's obvious that expected utility from cash should be the lowest. But from the results above, which stock should you invest in? (Fill in stock X, stock Y, or indifferent.)

[ ]

If you choose to invest in stock X, in the good state you will have a utility of 4.713486 (=ln(60+51.44)) and in the bad state you will have a utility of 4.183576 (=ln(30+35.60)). Therefore, your expected utility

from investing in X is 4.448531. Similarly, your expected utility from investing in Y is 4.448531 and from keeping it as cash is 4.426833. Your expected utility is the same from investing in either X or Y, therefore you are indifferent.

Capital Market Part 4

## Capital market - part 4

What if you have a linear utility function, e.g. $U = cw$, where $c$ is a constant and $w$ is your wealth. Now which stock do you prefer?

Hint: Are you now risk averse, risk seeking, or risk neutral? Compared to part 3? What about part 2? Try to use your logic and reasoning. You don't need to do any calculation for this question. (Of course you can brute-force it with calculation if you want to and I will give your full points if your calculation and conclusion are correct. )

To fully understand this, first go back to part 2 and 3. Notice that even though X has a higher return, you don't prefer it. This is because X positively correlates with your existing portfolio. In the good state, your ETF will do well anyway, and doing extraordinarily well on top of that don't mean much to you. In the meantime, in the bad state, your ETF will perform poorly and you really want other source of income to do well. But your investment in X will do poorly at the same time, so you suffer even more. All this is because you are risk-averse, i.e. you value the same amount of money more when you are poor compared to when you are rich. This is why you really love Y because it goes the opposite way compared to your existing portfolio (the ETF), serving as a hedge or insurance. This hedge/insurance value makes up for the lower expected return. That's why you are indifferent between X and Y even though X clearly has a higher expected return.

Now, if your utility function is linear, that means you are risk neutral and value the same amount of money exactly the same no matter you are rich or poor. This means the previous arguments about "not caring much when the ETF is doing well anyway" and "super miserable when everything goes wrong together" are no longer true. Therefore, Y no longer provides you hedge/insurance value. If the insurance value goes away, you will strictly prefer X because it simply has a higher return.

Lottery

## Lottery

You are offered to buy a lottery that has three tiers of prize. The jackpot gives you $10,000 with 0.1% chance; the good tier gives you $100 with 0.9% chance; and the okay tier gives you $5 with 99% chance. You have a log utility function $U = \ln(w)$ where $w$ is your wealth. Assume you currently don't have any other wealth. How much are you willing to pay for this lottery? That is, what is the certainty equivalence of this lottery. (Fill in a number with 2 decimal points.)

Hint: The probabilistically/actuarially fair price of this lottery is:

$$10000 \cdot 0.1\% + 100 \cdot 0.9\% + 5 \cdot 99\% = 15.85$$

This is not what this question is asking.

Your expected utility from this lottery is 1.644 (=ln(10000)*0.1%+ln(100)*0.9%+ln(5)*99%). Therefore your certainty equivalence is 5.18 (=e^1.644).

Portfolio Optimization Part 1

## Portfolio optimization - part 1

There are two stocks, DOG and CAT. DOG has an expected return of 18% with a standard deviation of 15%. CAT has an expected return of 12% with a standard deviation of 18%. The correlation of the returns of the two stocks is 0.2. The risk free rate is 2%. You cannot short sell any stocks. I optimized the risky portfolio for you: your optimal risky portfolio should consist 93.58% DOG and 6.42% CAT. What is the Sharpe ratio of the optimal risky portfolio? (Fill in a number with 4 decimal points.)

Also done to death. Expected return of your portfolio is 0.176148 and standard deviation is 0.143130. Therefore Sharpe ratio is 1.0910.

Portfolio Optimization Part 2

## Portfolio optimization - part 2

What's the optimal weight of DOG for the optimal risky portfolio if the correlation of the returns of the two stocks is 1? All other information remains the same compared to the last question. (Fill in a number with 2 decimals. Also notice that the percentage sign % is already written outside of the blank.)

The optimal risky portfolio should invest _____ % in DOG.

Notice that DOG has higher expected return with lower standard deviation compared to CAT, i.e. DOG is superior to CAT. In part 1 you want to still hold some CAT because it provides your diversification benefit. Now if the return correlation is 1, i.e. two stocks are perfectly positively correlated, CAT no longer provides any diversification benefit. Therefore you want to hold 100% of DOG given that you cannot short CAT.